ENTRANCE LENGTH IN TURBULENT FLOW OF GAS IN A CYLINDRICAL TUBE IN HIGHLY NONISOTHERMAL CONDITIONS

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We give theoretical and experimental data for the entrance length of a tube. The experiments were conducted in a range of enthalpy factor from 0.08 to 0.8 and Reynolds number from $6.9 \cdot 10^3$ to $2.4 \cdot 10^5$.

There have been a fairly large number of theoretical and experimental investigations of turbulent heat transfer and friction in the entrance section of a tube [1-9].

A theoretical investigation [1] showed that for a turbulent flow in tubes with a smooth entrance the entrance length is given by the equation

$$X_e = 0.693 R_0^{0.25}$$

From a numerical solution of the boundary-layer equation the authors of [2] obtained an approximate formula for the entrance length

$$X_e = 4.63 R_0^{0.12}$$

The authors of [3] assumed that the velocity in the potential core was constant along the entrance section for the cases T_ω=const and q_ω=const and obtained the following equations, respectively:

$$X_e = \frac{1}{A(m+1)} \left[\frac{3}{8} \frac{n}{2n+1}\right]^{m+1} \cdot R_0^m$$

$$X_e = \frac{1}{A} \left[\frac{3}{8} \frac{n}{2n+1}\right]^{m+1}, \qquad X = \frac{x}{D}$$

$$R_0 = \frac{\rho_{01} w_{01} D}{\mu_0}, \qquad \frac{cf_0}{2} = A(R^{++})^{-m}, \qquad \frac{cf_0}{2} = \frac{\tau_{w0}}{\rho_0 w_0^2}$$

The symbols used in the above equations are: x, D – tube length and diameter, respectively; ρ – density; ω – velocity; μ – viscosity; T – temperature; q – specific heat flux; τ – tangential stress; δ^{++} – momentum thickness; n – index of power in law of velocity distribution in boundary layer. The subscripts are: ω – parameters on wall; 0 – parameters in flow core, and e – parameters on boundary of entrance section.

In the experimental investigations [4, 5] (Table 1) the entrance length was determined from the change in the local Nusselt number Nu and the local heat transfer coefficient. The cross section of the tube where these data exceeded their asymptotic values by a particular amount (5% or 1%) was taken as the start of the stabilized section. This method cannot give satisfactory results, since it fails to take into account the special features in the entrance section. Owing to the increase in thickness of the boundary layer and increase in the velocity in the flow core the heat transfer coefficient decreases at first and then increases again [4, 6].

The methods of determination were: A_1 – from the change in the local heat transfer coefficient; A_2 – from the change in the local pressure gradient; A_3 – from a comparison of the stagnation enthalpy and

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TABLE 1

Liquid	Range of R	x_e	Method	Experimental conditions	Reference
Water	$1.7 \cdot 10^4 - 9 \cdot 10^4$	10—15	A_1	$q_w = \text{const}$	[4]
Air	$2.7 \cdot 10^4$	12	A_1	$T_w = \text{const}$	[4]
Air, carbon dioxide gas	$5.10^4 - 2.5 \cdot 10^5$	11—27	A_1	$q_w = \text{const}$	[5]
Water	$10^4 - 10^6$	40—20	A_1	$T_w \doteq \text{const}$	[8]
Water	$4.9 \cdot 10^4 - 6.5 \cdot 10^4$	14—20	A_2		[6]
Air	$3 \cdot 10^3 - 4.2 \cdot 10^3$	16—17	A_3		[7] ‡
Air	$6.9 \cdot 10^3 - 2.4 \cdot 10^5$	7.9—22	A_4	$T_w = \text{const}$	

*The length of the hydrodynamic entrance section was determined. The data are taken from an analysis of the "5%" graph in Fig. 7.
†The thermal and hydrodynamic entrance lengths were determined.
‡Authors of this paper.

velocity calculated for the entrance and main sections; and A_4 – from the change in enthalpy on the tube axis.

The values of X_e obtained in [7] by comparison of the stagnation enthalpy and the velocity on the axis in the entrance and main sections are probably overestimated in comparison with the other data given in the table.

The table indicates that there are great differences between the data of different investigations even for the case of quasi-isothermal flow. In particular, it follows from [8] that the entrance length decreases with increase in the Reynolds number, which contradicts the results of [1-7]. The effect of intense cooling of the wall on the thermal entrance length has hardly been investigated at all.

Below we give the results of a theoretical and experimental determination of the thermal entrance length in significantly nonisothermal conditions at subsonic gas flow velocities.

For the entrance section in a subsonic flow with h_ω=const and Ψ_h=const, a joint solution of the momentum equation

$$\frac{dR^{++}}{dX} + \frac{R^{++}}{W_0}\frac{dW_0}{dX}(1+H) R_1 \Psi \frac{cf_0}{2}$$

the continuity equation

$$R^{++} = \frac{R_1 (W_0 - 1)}{4H} \tag{1}$$

and the friction law $\frac{1}{2} cf_0 = 0.0128\, R^{**-m}$ gives [9]

$$\left[\frac{5}{4}(1+H)+1\right]\left[4(W_0-1)^{0.25} - \frac{1}{\sqrt{2}} \ln \frac{(W_0-1)^{0.5} + \sqrt{2}(W_0-1)^{0.25}+1}{(W_0-1)^{0.5} - \sqrt{2}(W_0-1)^{0.25}+1} - \right.$$
$$\left. - \sqrt{2} \operatorname{arc\,tg} \frac{\sqrt{2}(W_0-1)^{0.25}}{1-(W_0-1)^{0.5}}\right] - (1+H)\frac{(W_0-1)^{1.25}}{W_0} = \Psi \frac{0.0725 H^{1.25}}{R_1^{0.25}} X$$
$$R^{++} = \frac{\rho_0 w_0 \delta^{++}}{\mu_w}, \quad H = \frac{\delta^+}{\delta^{++}}, \quad R_1 = \frac{\rho_{01} w_{01} D}{\mu_w}$$
$$\Psi = \frac{cf}{cf_0}, \quad W_0 = \frac{w_0}{w_{01}}, \quad \psi_h = \frac{h_w}{h_0} \tag{2}$$

Here and henceforth δ^+ is the displacement thickness, h is the enthalpy of the gas, H_0 is the shape factor in isothermal conditions for a gradient-free flow, i.e., $dP/dx=0$; the subscript 1 denotes parameters at the tube entrance; the subscript h denotes parameters for a given enthalpy factor. The other symbols are as before.

At the end of the entrance section the thickness of the boundary layer becomes equal to the tube radius (r_0). Then it follows from Eq. (1) ($\rho_0 \approx \rho_{01}$) that

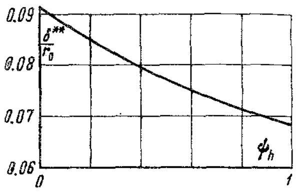

Fig. 1

$$W_{0H} = \left(1 - 2\,\frac{\delta^{++}}{r_0}H\right)^{-1} \tag{3}$$

where W_{0e} is the relative velocity W_0 at the end of the entrance section.

The value of δ^{++}/r_0 can be found as a function of Ψ_n by assuming similarity of the velocity and total enthalpy distributions

$$\frac{w}{w_0} = \frac{h - h_w}{h_0 - h_w}$$

According to [9], for a subsonic gas flow

$$\frac{\rho_0}{\rho} = \psi_h - (\psi_h - 1)\,\frac{w}{w_0}$$

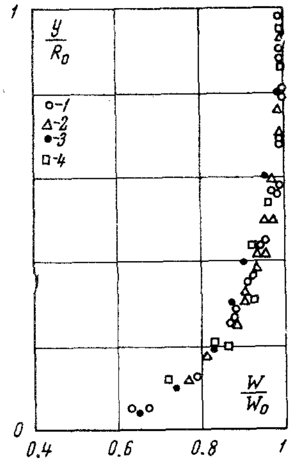

Fig. 2

Figure 1 shows the relationship $\delta^{++}/r_0 = f(\Psi_n)$ for $\omega/\omega_0 = \xi^{1/7}$. Figure 2 shows the results of measurement of the velocity profile at the end of the tube for different degress of nonisothermality. The velocities were measured by tracing the flow with fine aluminum powder [10]. Points 1 show Pitot tube measurements, points 2 measurements by the tracing method, and points 3 show Nikuradze's data [11] for $R_0 = 1.1 \cdot 10^5$, corresponding to $\Psi_n = 1$. Points 4, measured by the tracing method, correspond to $\Psi_n = 0.2$. As the graph shows, the nonisothermality has no appreciable effect on the turbulent part of the velocity profile. A similar result was obtained in [7].

According to [12],

$$\Psi = \left(\frac{2}{\sqrt{\psi_h} + 1}\right)^2 \qquad \text{if} \quad cf_0 = f\left(\frac{w_0 \rho_0 \delta^{++}}{\mu_w}\right) \tag{4}$$

The shape factor H for subsonic flow, assuming $R^{++} \approx R_h^{++}$, can be calculated with sufficient accuracy as [9]

$$H = H_0 \psi_n, \quad H_0 = 1.347 \tag{5}$$

Joint solution of (2)-(5) gives the required relationship

$$\frac{X_H}{R_1^{0.25}} = f(\psi_h)$$

which can be approximated by the equation

$$\frac{X_H}{R_0^{0.25}} = \frac{0.8\psi_h + 0.55}{\psi_h^{0.25\,k}}, \qquad R_0 = \frac{4G}{\pi D \mu_0} \quad \text{for} \quad \frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^k \tag{6}$$

To verify these results we carried out some experiments. A diagram of the experimental apparatus is shown in Fig. 3, where 1 is a plasma generator, 2 is a mixing chamber, 3 is the test section, 4 is a measuring orifice, 5 is a mixer, 6 and 10 are thermocouples, 7 measures the static pressure 8 measures the water temperature at the outlet of the calorimeter, 9 is a measuring vessel, 11 measures the wall temperature, 12 is the air supply, 13 is the inflow of cooling water, 14 measures the water temperature at the entrance, and 15 is heat insulation.

The air was heated by three water-cooled dc plasma generators with vortical and magnetic rotation of the arc. The power depended on the number of operating plasma generators and sectional rheostats in the ac circuit. The heated air from the plasma generators entered a common, water-cooled, mixing chamber. As measurements with cold air showed, the velocity profile at the outlet of the chamber was fairly uniform. This can be attributed to the great convergent effect of the mixing chamber (100:1) and to

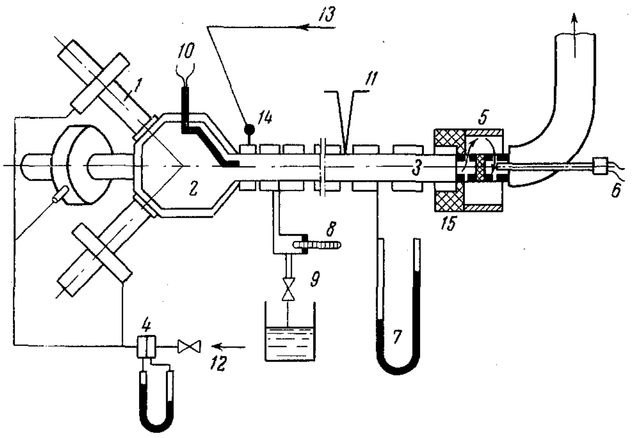

Fig. 3

the fact that the axes of the rotating streams from the plasma generators meet at one point.

The test section was a continuous copper tube with internal diameter 18.5 mm and wall thickness 1.5 mm. A copper tube 5·0.5 mm was fitted in separate sections to the outside of the main tube with PS-72 solder to form 14 calorimeters of different length (X=0.865, 1.73, 2.65, 3.84, 5.46, 8.11, 11.62, 15.62, 20.11, 25.62, 31.56, 37.84, 44.49, 51.24; X is the length measured from the start of the tube). The calorimeters were cooled with water from a constant-level tank. The water flow rate was measured by the volume method and its temperature was measured with mercury thermometers with a scale division of 0.5° C and Δt=30° C. The tube wall temperature was measured with chromel-copel thermocouples of diameter 0.1 mm, embedded in the gaps between the section. In these cross sections the static pressures were sampled.

The flow of working air was measured with orifices. The mean temperature of the air was measured at the outlet of the working section with a platinum-platinum-rhodium thermocouple with the aid of a porcelain mixer, which also served as a heat shield for the thermocouple. The enthalpy at the entrance to the tube was determined by calculation from the known air temperature at the outlet and the total heat removal from the experimental section. This method was tested experimentally in all the experiments in which the temperature T_{01} at the entrance was less than 1600° C. The measured and calculated values of the temperature T_{01} practically coincided in every case. This also indicated that the temperature profile at the entrance to the tube was fairly uniform. A provisional estimate of the heat loss showed that it was less than 1%.

The experiments were conducted only in steady-state conditions. The voltage fluctuation on the plasma generator did not exceed ±1%. The experiments were conducted in the range

$$\psi_h = 0.8 - 0.08,$$
$$R_0 = 6.9 \cdot 10^3 - 2.4 \cdot 10^5$$

The entrance length was determined in the following way.

The energy balance equation for a section of tube of length x can be put in the form

$$\rho_{01} w_{01} h_{01} r_0^2 = 2\int_0^x q_w r_0 dx + \int_0^{r_0} 2\rho w h r\, dr \tag{7}$$

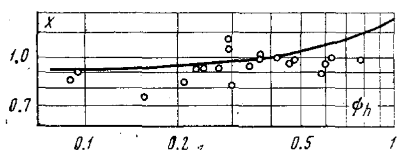

Fig. 4

Putting $\delta_h^{++} = \delta^{++}$ (δ_h^{++} is the energy thickness) and assuming for the main section of the tube the equality

$$\rho_0 w_0 = \frac{\rho_{01} w_{01}}{1 - 2H\delta^{++} / r_0}$$

we obtain from (7) by using (5)

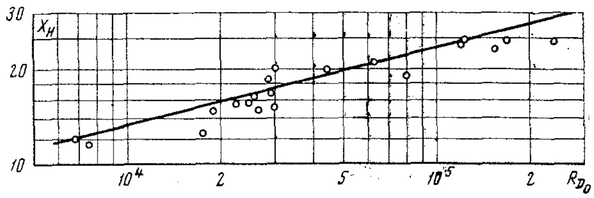

Fig. 5

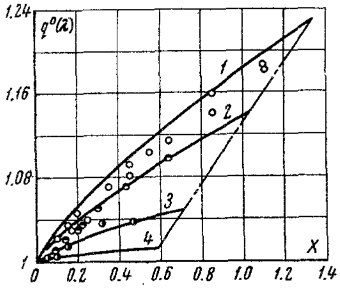

Fig. 6

$$\frac{h_{01}-h_{wi}}{h_0-h_{wi}}=\frac{1-W}{1-S} \tag{8}$$

Here

$$N=\frac{1}{r_0/2\delta^{++}-H_0\psi_h}, \qquad S=\frac{1}{G(h_{01}-h_{wi})}\sum_1^n Q_{wi}$$

$Q_{\omega i}$ is the amount of heat received by the i-th calorimeter, and G is the gas flow rate through the tube.

At the end of the entrance length we must obviously have

$$\frac{h_{01}-h_{wi}}{h_0-h_{wi}}=1$$

In this case it follows from (8) that

$$N=S \tag{9}$$

The calculation was performed by an M-20 computer. The cross section of the tube for which equality (9) was satisfied to within $\pm 1\%$ was taken as the end of the thermal entrance section. The calculation was made from the curve of $q_{\omega i}=f(X)$ found as the root-mean-square of the experimental values of $q_{\omega i}$ from the end of the tube to its beginning. The value of δ^{++}/r_0 was found from the relationship in Fig. 1.

Figure 4 shows the experimental values of $X_e/R_0^{0.25}$ in relation to the enthalpy factor for k = 0.64. The curve shown on this graph, which corresponds to the theoretical solution [Eq. (6)], agrees satisfactorily with the experimental results.

In Fig. 5 these experimental data are reduced to quasi-isothermal conditions from (6) and are represented in the form of the relationship $X_e=f(R_0)$. The continuous line on this graph corresponds to Eq. (6) for $\Psi_h=1$:

$$X_H=1.35R_0^{0.25}$$

Figure 6 shows the change in the relative velocity, found theoretically from Eq. (2), in the entrance section for various values of Ψ_h; curves 1, 2, 3, and 4 correspond to Ψ_h=1.0, 0.6, 0.2, and 0.044.

The measured velocities shown on the same graph agree satisfactorily with theory.

From this investigation we can draw the following conclusions.

1. The entrance length for a gas flow increases significantly with increase in the Reynolds number at the tube entrance and is given fairly satisfactorily by Eq. (6).

2. Cooling of the tube has a relatively smaller effect on the entrance length.

In the range of Ψ_h from 1 to 0.08 the entrance length is reduced by only 30%.

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